

Chapter 5—Probability

5.1 Probability of Simple Events

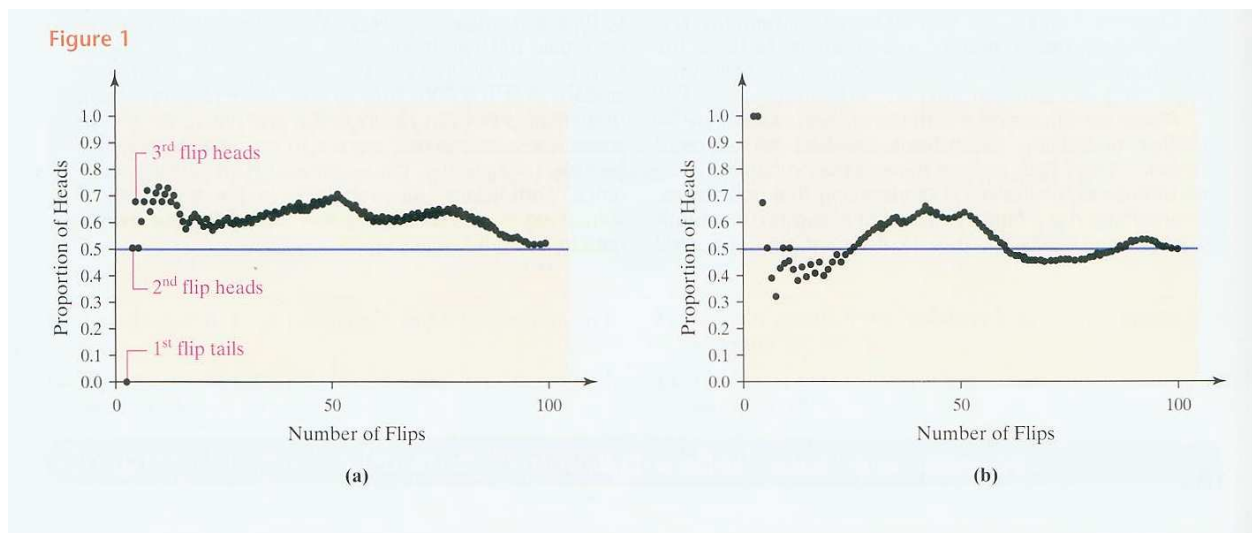
Probability is the likelihood (or chance) of an outcome or event (p. 223).

Experiment—a process that can be repeated and the results are uncertain.

E.g., tossing a fair coin is an experiment.

$$P(\text{head}) = \frac{1}{2} \quad \text{where head is an event.}$$

Note: (1) Probability describes the long-term proportion with which an event will occur.
(2) Probability does not perfectly predict the short-run outcome.



Law of Large Numbers: As the number of repetitions of a probability experiment increases, the proportion with which a certain outcome is observed gets closer to the probability of the outcome.

- For a fair coin, the TRUE probability of a head is 0.50.
- However, the proportion of heads in a given experiment will not typically equal 50%.
- In the long run, after many repetitions of the experiment, the proportion of heads will approach 50%.

Experiment is a process that can be repeated and the results are uncertain.

Simple event (e_i)—outcome from a single trial of the experiment.

Sample space (S)—collection of ALL possible simple events.

Event (E)—any collection of outcomes from an experiment. An event can consist of one or more simple events.

E.g., tossing a single “fair” die is an experiment.

Simple events: $e_1=1$

$e_2=2$

$e_3=3$

$e_4=4$

$e_5=5$

$e_6=6$

Sample space: $S = \{1,2,3,4,5,6\}$

Event: $E = \text{“roll an even number”} = \{2,4,6\}$

Properties of Probabilities (p. 225):

(1) $0 \leq P(E) \leq 1$

(2) $P(\text{impossible event})=0$

(3) $P(\text{certain event})=1$

(4) The probability of ALL simple events in the sample space equals 1.

If $S=\{e_1, e_2, \dots, e_n\}$, the $P(e_1)+P(e_2)+\dots+P(e_n)=1$.

Three Methods for Determining the Probability of an Event:

(1) Classical

(2) Empirical

(3) Subjective

Classical Method—requires equally likely outcomes, i.e., each simple event has the same probability of occurring.

Computing Probabilities Using the Classical Method (p. 227):

$$P(E) = \frac{N(E)}{N(S)},$$

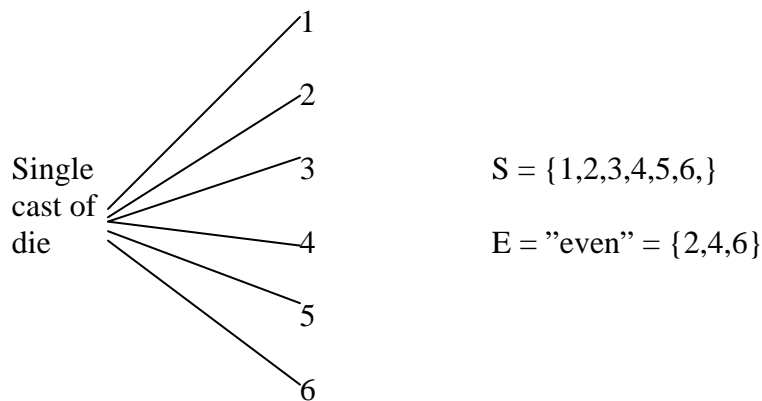
where E=event

S=sample space of ALL simple events

N(E)=number of simple events in E.

N(S)=number of simple event in S.

Example: A single die is cast. What is the probability of an even?



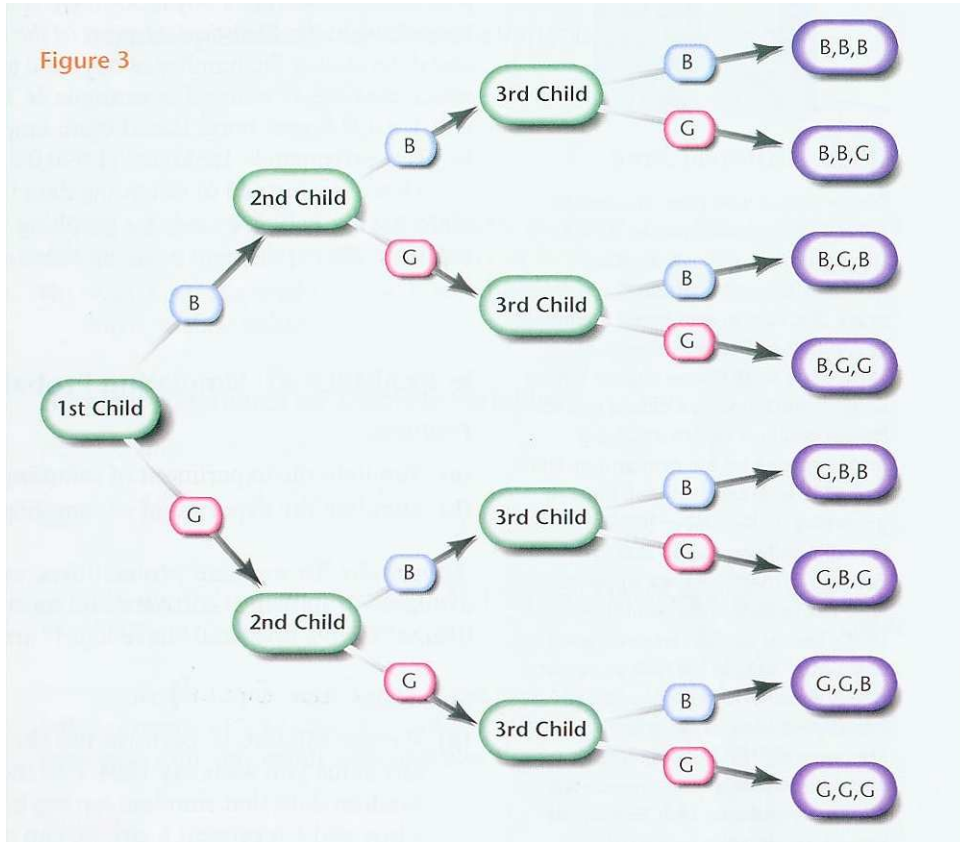
Can the Classical Method be applied in this situation?

What is the probability of an even?

$$P(E) = \frac{N(E)}{N(S)} = \frac{3}{6} = \frac{1}{2} = 0.50$$

If the die cast is repeated 100 times, will we get 50 evens? Justify your answer.

Example 7, p. 229—For a family with three children, what is the probability of two boys and one girl?



$S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\} \gggg N(S)=8$

$E = \text{"2 boys + 1 girl"} = \{BBG, BGB, GBB\} \gggg N(E)=3$

$$P(E) = \frac{N(E)}{N(S)} = \frac{3}{8} = 0.375 = 37.5\%$$

Empirical Method:

Approximating Probabilities through the Empirical Approach

The probability of an event E is approximately the number of times event E is observed divided by the number of repetitions of the experiment.

$$P(E) = \frac{\text{frequency of } E}{\text{number of trials of experiment}}$$

Empirical probability is the relative frequency as defined in a frequency distribution (pp. 56, 72, and 74 Text).

E.g., Fifty families with three children are surveyed to determine the number of boys and girls in the family.

Event	Frequency	Relative Frequency
3 boys	7	0.14
3 girls	6	0.12
1 boy + 2 girls	19	0.38
2 boys + 1 girl	<u>18</u>	<u>18/50=0.36</u>
	50	1.00

Compare the Empirical Probability above to the Classical Probability and explain why they differ.

Subjective Probabilities:

Definition: Subjective probabilities are probabilities based upon an educated guess.

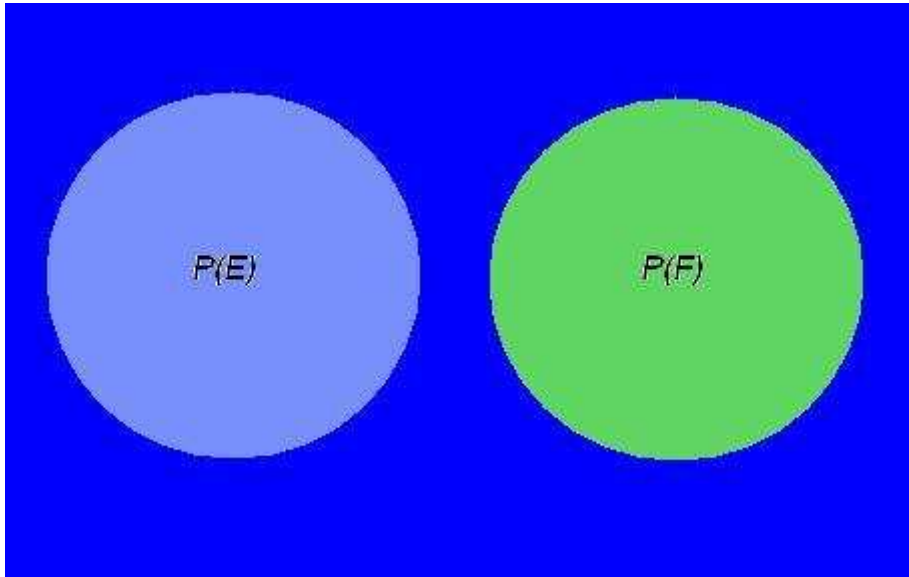
Example: What is the probability that the economy will fall into a recession next year?

- The first economist says the probability is 0.25.
- The second economist says the probability is 0.50.
- The third economist says the probability is 0.10.

How can three well-trained economists have such different opinions about the probability of a recession? Because the probabilities they stated are “educated guesses” based upon information they currently have available.

The Addition Rule (from Section 5.2)

Mutually Exclusive or Disjoint Events—are unique events (i.e., events E and F cannot occur simultaneously).



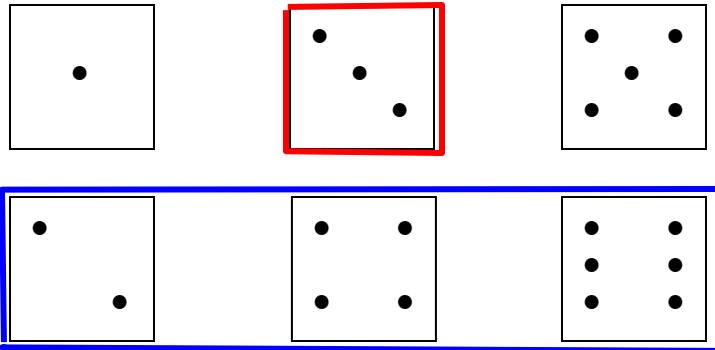
Example: In a coin toss, the events head and tail are mutually exclusive..

Addition Rule for Mutually Exclusive Events: If E and F are mutually exclusive events, then:

$$P(E \text{ or } F) = P(E) + P(F)$$

Example of the Addition Rule for Mutually Exclusive Events:

Toss a single “fair” die. What is the probability of an even or a 3?



$$\begin{array}{l}
 E = \text{“roll an even”} = \{2,4,6\} \gggg N(E) = 3 \\
 F = \text{“roll a 3”} = \{3\} \gggg N(F) = 1
 \end{array}$$

So, the probability of E or F is:

$$\begin{aligned}
 P(E \text{ or } F) &= P(E) + P(F) \\
 &= \frac{N(E)}{N(S)} + \frac{N(F)}{N(S)} \\
 &= \frac{3}{6} + \frac{1}{6} \\
 &= \frac{4}{6}
 \end{aligned}$$

With **OR** and **mutually exclusive** events, you add probabilities.

The Multiplication Rule (from Section 5.3)

Definition: Two events E and F are **independent** if the occurrence of event E in a probability experiment does not affect the probability of event F. Two events are **dependent** if the occurrence of event E in a probability experiment affects the probability of event F.

Example of Independent and Dependent Events: Is rainfall in Lubbock independent of rainfall in NY?—YES.

Is rainfall in Lubbock independent of rainfall in Brownfield?—NO. Rainfall in Lubbock and Brownfield are dependent (because of the close distance between the two cities).

Multiplication Rule for Independent Events:

If E and F are independent events, the probability that E and F both occur is:

$$P(E \text{ and } F) = P(E) * P(F)$$

In words, the probability of E and F is the probability of event E times the probability of event F.

Example of Independent Events: Toss a fair coin two times. What is the probability of a head on the 1st toss and a head on the 2nd toss?

To determine whether we can use the Multiplication Rule for Independent Events, we must first determine whether the outcome of the two tosses are independent.

Does the probability of a head on the 1st toss affect the probability of a head on the 2nd toss?—NO. Thus the two events H1 and H2 are independent.

Now that we have established that the two events H1 and H2 are independent, the Multiplication Rule for Independent Events can be applied to the question of the probability of a head on the 1st toss and a head on the 2nd toss.

$$\begin{aligned} P(H1 \text{ and } H2) &= P(H1) * P(H2) \\ &= \frac{1}{2} * \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

With **AND** plus **independent events**, you multiply probabilities.

The Factorial Symbol and Combinations (from Section 5.5)

Definition: If $n > 0$ is an integer, the **factorial symbol** $n!$ is defined as follows:

$$\begin{aligned} 0! &= 1 & 1! &= 1 \\ n! &= n*(n-1)* \dots *3*2*1 \end{aligned}$$

For example, $2! = 2*1 = 2$, $3! = 3*2*1 = 6$, $4! = 4*3*2*1 = 24$, and so on.

Definition: A **combination** is a collection, without regard to order, of n distinct objects without repetitions. The symbol nCr represents the number of combinations of n distinct objects taken r at a time.

The number of different arrangements of n objects using $r \leq n$ of them, in which

1. the n objects are distinct
2. once an object is used, it cannot be repeated
3. order is not important

Is given by the formula

$$nCr = \frac{n!}{r!(n-r)!}$$

Problem: Fred, John, and Tom are going to play doubles tennis against a rival team. They need to randomly select a doubles team of two people. List all possible team combinations. That is, list all the combinations of the three people Fred, John, and Tom taken two at a time. What is ${}_3C_2$?

$${}_3C_2 = \frac{3!}{2!(3-2)!} = \frac{3*2!}{2!*1!} = \frac{3}{1} = 3$$

List the 3 combinations: Fred and John; Fred and Tom; John and Tom.

Problem: Assume we have two events S and F and that S occurs k times and F occurs $(n-k)$ times. There are nCk ways of arranging the selections of k S 's and $(n-k)$ F 's. For example, assume you have 2 S 's and 1 F . The combination formula, nCk , can be used to determine the number of arrangements:

${}_3C_2 = 3$ with arrangements

_____	_____	_____
_____	_____	_____
_____	_____	_____

Problem: Assume you have 2 S's and 2 F's. How many arrangements are there of 2 S's and 2 F's? List the arrangements.

$${}^4C_2 = \frac{4 * 3 * 2!}{2! * (4 - 2)!} = \frac{4 * 3}{2!} = \frac{12}{2} = 6$$

The arrangements are:

S	S		
S		S	
S			S
	S	S	
	S		S
		S	S